Correction and Extension of the Concept of Cross-Spin Control

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Nidey has shown,[1] albeit incorrectly, that the cross-spin of a symmetrical rigid body can be easily controlled independently of the spin by rotating a pair of control jets about the axis of symmetry in synchronism with the cross-spin. Let $\boldsymbol{\lambda}$ and $\boldsymbol{\tau}$ be defined as unit vectors parallel and normal to the axis of symmetry, respectively, oriented so that $\boldsymbol{\omega}$, the angular velocity of the body, has only two components, $\omega_{\lambda,\tau}$ (the spin and cross-spin, respectively). In this coordinate system the angular momentum of the symmetrical body also has only two components:

$$\mathbf{H} = I_\lambda \omega_\lambda \boldsymbol{\lambda} + I_\tau \omega_\tau \boldsymbol{\tau} \quad (1)$$

where $I_{\lambda,\tau}$ are the longitudinal and transverse moments of inertia, respectively. Thus

$$\dot{\mathbf{H}} = I_\lambda \dot{\omega}_\lambda \boldsymbol{\lambda} + I_\tau \dot{\omega}_\tau \boldsymbol{\tau} + I_\lambda \omega_\lambda (\boldsymbol{\Omega} \times \boldsymbol{\lambda}) + I_\tau \omega_\tau (\boldsymbol{\Omega} \times \boldsymbol{\tau}) \quad (2)$$

where $\boldsymbol{\Omega}$ is the angular velocity of the coordinate system relative to an inertial reference. But $\boldsymbol{\Omega}$ must share the transverse component of the angular velocity of the body, i.e.,

$$\boldsymbol{\Omega} = \Omega_\lambda \boldsymbol{\lambda} + \omega_\tau \boldsymbol{\tau} \quad (3)$$

Hence

$$M_\lambda = I_\lambda \dot{\omega}_\lambda \quad (4)$$

$$M_\tau = I_\tau \dot{\omega}_\tau \quad (5)$$

and

$$M_\nu = I_\tau \omega_\tau \Omega_\lambda - I_\lambda \omega_\lambda \omega_\tau \quad (6)$$

where $\boldsymbol{\nu} \equiv \boldsymbol{\lambda} \times \boldsymbol{\tau}$ and $M_{\lambda,\tau,\nu}$ are the components of the external moment. Fortunately, the primary conclusion of Ref. 1 is confirmed: the component of the external moment parallel to the cross-spin (transverse component) modifies the cross-spin directly independently of spin. Furthermore, the component normal to the cross-spin and to the axis of symmetry (normal component) produces rotation of the cross-spin. The rate of rotation is given by Eq. (6):

$$\Omega_\lambda = \Omega_0 + M_\nu/(I_\tau \omega_\tau) \quad (7)$$

where $\Omega_0 \equiv (I_\lambda/I_\tau)\,\omega_\lambda$ = const for a free body[2] as well as for one subjected to a control moment, $\mathbf{M}_c$, perpendicular to the axis of symmetry.

Let β be the angle between $\mathbf{M}_c$ and the cross-spin; the transverse and normal components are then $M_\tau = M_c \cos\beta$ and $M_\nu = M_c \sin\beta$, respectively. If the cross-spin is to be increased, $-\pi/2 < \beta < \pi/2$; or to be decreased, $\pi/2 < \beta < 3\pi/2$. If $0 < \beta < \pi$, the normal component is positive, $\Omega_\lambda > \Omega_0$ and β tends to be decreased, aligning the cross-spin vector with the control moment; whereas, if $\pi < \beta < 2\pi$, the normal component is negative, $\Omega_\lambda < \Omega_0$ and β tends to be increased, again aligning the cross-spin vector with the control moment. Hence, the rotation of the cross-spin produced by the normal component is always of such a sense as to tend to synchronize the cross-spin with the transverse component. Thus it is easy to increase and difficult to reduce the cross-spin. Specifically, no rotation of the control jet is required if the cross-spin is to be increased; whereas very accurate servo-control of the jet is required if the cross-spin is to be reduced to a very small value.

To illustrate these conclusions consider the Aerobee sounding rocket for which $I_\lambda = 4.8$ slug-ft², $I_\tau = 450$ slug-ft², and $\omega_\lambda = 2.0$ rad/sec are reasonably characteristic values. Assume that a control moment provided by a 10 lb thrust jet located 5 ft from the center of mass is to be used for the cross-spin control; i.e., $M_c = 50$ lb-ft. For $\beta < 45°$, for which the jet is still 70% efficient in increasing the cross-spin, ω_τ can be as great as

$$\begin{aligned}\omega_\tau &= M_c \sin\beta/(I_\tau \Omega_\lambda - I_\lambda \omega_\lambda) \\ &< [(50)(0.707)]/[(450)(2.0) - (4.8)(2.0)] \\ &< 3.97 \times 10^{-2}\ \text{rad/sec}\end{aligned} \quad (8)$$

Thus, the free-body precession half-angle θ could be made as large as

$$\begin{aligned}\theta &= \arctan[(I_\tau\omega_\tau)/(I_\lambda\omega_\lambda)] \\ &< \arctan\{[(450)(3.97 \times 10^{-2})]/[(4.8)(2.0)]\} \\ &< 62°\end{aligned} \quad (9)$$

in less than one-half second since the time increment, Δt, is given by the expression

$$\begin{aligned}\Delta t &= [(\Delta\omega_\tau)(I_\tau)]/[M_c \cos\beta] \\ &< [(3.97 \times 10^{-2})(450)]/[(50)(0.707)] \\ &< 0.51\ \text{sec}\end{aligned} \quad (10)$$

Contrariwise, if the cross-spin is to be reduced to 3.97×10^{-5} rad/sec corresponding to a precession half-angle of 6 min of arc, the required accuracy of control moment alignment would be

$$\begin{aligned}\beta &= \arcsin[\omega_\tau (I_\tau\Omega_\lambda - I_\lambda\omega_\lambda)/M_c] \\ &< \arcsin\{(3.97 \times 10^{-5})\,[(450)(4\pi + 2.0) - (4.8)(2.0)]/50\} \\ &< 15\ \text{min of arc}\end{aligned} \quad (11)$$

if the jet is not to be rotated more than 2 rps relative to the rocket body. By reducing M_c this limit, of course, could be increased correspondingly.

In any event, it is clear that cross-spin control can be of use in the space astronomy program and that the implementation of practical control systems either to increase or to decrease the cross-spin without first despinning the vehicles can be devised; though the latter is considerably more difficult.

References

[1] Nidey, R. A., "Unique cross-spin control concept for fixed fin sounding rockets," ARS J. **31**, 824 (1961).

[2] Thomson, W. T., *Introduction to Space Dynamics* (John Wiley and Sons, Inc., New York, 1961), Chap. 5, p. 116.

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